Math 270: Differential Equations Day 9 Part 2

Section 4.2: Solving Second-Order Linear Homogeneous DEs Where The Auxiliary Equation Has Only Real Roots

Section 4.2: Solving Second-Order Linear Homogeneous DEs With Constant Coefficients Where The Auxiliary Equation Has Only Real Roots

Ex: Find a solution to $ay'' + by' + cy = 0$ of the form $y = e^{rx}$

Conclusion:

1) If r_1 is a root of the auxiliary equation $ar^2 + br + c = 0$, then y=$e^{r_1 x}$ is a solution to $ay'' + by' + cy = 0$

2) If r_1 and r_2 are two distinct real roots of the auxiliary equation $ar^2 + br + c = 0$, then y=$e^{r_1 x}$ and y=$e^{r_2 x}$ are 2 independent solutions to $ay'' + by' + cy = 0$ and hence y=$Ae^{r_1 x} + Be^{r_2 x}$ is the general solution

Section 4.2: Solving Second-Order Linear Homogeneous DEs With Constant Coefficients Where The Auxiliary Equation Has Only Real Roots

Ex 1: Find the general solution to $y'' - 3y' - 10y = 0$

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Ex 2: Find the general solution to $3y'' - 5y' + y = 0$

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Ex: Find a solution to $ay'' + by' + cy = 0$ of the form $y = e^{rx}$

Conclusion:

1) If r_1 is a root of the auxiliary equation $ar^2 + br + c = 0$, then y=$e^{r_1 x}$ is a solution to $ay'' + by' + cy = 0$

2) If r_1 and r_2 are two distinct real roots of the auxiliary equation $ar^2 + br + c = 0$, then y=$e^{r_1 x}$ and y=$e^{r_2 x}$ are 2 independent solutions to $ay'' + by' + cy = 0$ and hence y=$Ae^{r_1 x} + Be^{r_2 x}$ is the general solution

Question: What if *r* is a repeated root?

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Ex: Show that if r_1 is a root of the auxiliary equation $ar^2 + br + c = 0$ of multiplicity 2, then $y = e^{r_1 x}$ and $y = xe^{r_1 x}$ are 2 independent solutions of $ay'' + by' + cy = 0$

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To Solve A Second-Order Linear Homogeneous DE With Constant Coefficients Where The Auxiliary Equation Has Only Real Roots: Solving $ay'' + by' + cy = 0$

1) Find the roots of the auxiliary equation $ar^2 + br + c = 0$

2) If r_1 and r_2 are two distinct real roots of the auxiliary equation $ar^2 + br + c = 0$, then y=$e^{r_1 x}$ and y=$e^{r_2 x}$ are 2 independent solutions to $ay'' + by' + cy = 0$ and hence y=$Ae^{r_1 x} + Be^{r_2 x}$ is the general solution

3) If r_1 is a real root of multiplicity 2 of the auxiliary equation, then y=$e^{r_1 x}$ and y=$xe^{r_1 x}$ are 2 independent solutions to $ay'' + by' + cy = 0$ and hence y=$Ae^{r_1 x} + Bxe^{r_1 x}$ is the general solution

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Ex 3: Find the solution to the IVP $y'' + 6y' + 9y = 0$, $y(0) = 1$, $y'(0) = 3$

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Ex 4:
The auxiliary equation for $y^{(6)} + 4y^{(5)} - 50y^{(4)} - 20y^{(3)} + 256y'' + 376y' + 144y = 0$
is $r^6 + 4r^5 - 50r^4 - 20r^3 + 256r^2 + 376r + 144 = 0$
or $(r+1)^3(r-4)^2(r+9) = 0$
Find the general solution to this DE.

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Ex 5 (Book Section 4.2 ex 4): Find the general solution to $y''' + 3y'' - y' - 3y = 0$